INVESTIGATION OF THE STATISTICAL DECISION PROCESS FOR ANTI-SUBMARINE WARFARE TACTICAL DECISIONS

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Submitted in partial fulfillment of
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ABSTRACT

The past few years have brought an increased interest in the scientific approach to decision-making. Current literature generally concerns itself with analytical theories.

This paper investigates an application of the statistical decision process to the problem of ASW tactical decisions. The Bayesian decision process is utilized.

The paper analyzes the basic ASW decision problem with emphasis on the uncertainty aspect of a possible submarine contact. A mechanism is developed to formally connect the general problem areas.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction.

A large amount of literature is presently available on the theory of decision-making, and investigation of the subject continues. Contemporary interest in the analysis of the decision-making process is generally attributed to the published work of von Neumann and Morgenstern. [12] The majority of the literature concerns itself with analytical theories of the process. A few writers have attempted to apply the theory to specific real world decision problems. [6,7]

One of the purposes of postgraduate education at the U. S. Naval Postgraduate School is to provide the student with sufficient scientific and technical background to permit him to fill the middle-ground between the scientist and the naval officer -- to provide the capability of interpreting present technical theory and development with an eye toward its application to naval warfare. This paper is an exercise in such interpretation. It is an investigation of the current literature on the decision-making process with a specific application to the decision problems of an anti-submarine warfare commander.

This paper attempts to analyze the ASW commander's tactical decision problems. The objective of the commander's analysis is to select a particular course of action, from among the available courses of action, that is consistent with the commander's desire for a particular result.

The heart of the problem is the uncertainty associated with a "possible submarine contact"; an indication from one or more detection sensors that a submarine is present. Is it a submarine or isn't it?

This uncertainty is currently being evaluated by the commander and his staff on the basis of experience and a particular "feel" for the ASW

tactical situation. The purpose of the analysis in this paper is to provide a formalism for taking into account the commander's preferences and the degree of contact uncertainty, rather than leaving it to the decision-maker's unaided "feel" for the problem.

An effort has been made to avoid the extreme technical terminology and mathematical theory prevalent in much of the academic and theoretical expositions of the subject. An attempt is made to confine the discussion of theory to relevant areas of interest to the military reader. Although an ASW Hunter-Killer Group situation is used as an illustrative vehicle in investigating the decision problem, the analysis is equally applicable to maritime patrol aircraft and surface escort operations.

This paper is an analysis of the basic decision problem with emphasis on the uncertainty aspect of a possible submarine contact. A mechanism is developed to formally connect the general aspects of the problem. The Bayesian decision approach is utilized.

2. Factors Influencing the ASW Commander's Decision

The generation of a possible submarine contact in anti-submarine warfare operations will pose various decision requirements for an ASW commander. Among the decisions to be made are, whether or not to prosecute the contact, whether or not to provide additional forces, and whether or not to expend weapons.

Inherent in the criteria for these decisions is the weight the decision-maker places on the consequences of 1) classifying a non-submarine contact as submarine, 2) classifying a submarine contact as non-submarine, 3) classifying a non-submarine contact as non-submarine, and 4) classifying a submarine contact as submarine. The consequences of each alternative classification will vary with the particular tactical situation. For instance, classifying a valid submarine contact as non-submarine when defending a continent against missile launching submarines would be much more critical than making the same classification in a force-support situation with the contact fifteen miles astern of a fast moving carrier task force.

Inputs for these decisions include forces and weapons available, the number of contacts currently being prosecuted, and the prospects of additional valid contacts.

The solutions to these decision problems are typically based on the "feel" that a commander and his staff have for the anti-submarine warfare problem. This "feel" is primarily the result of past experience. The increasing distance in time since World War II, coupled with the increased complexity and sophistication of airborne, surface, and subsurface ASW equipments dictates that this experience be gained during

peacetime training exercises.

The inborn artificialities of exercise situations tend to obscure many of the crucial inputs that must necessarily be considered in a wartime situation. As an example, training exercises span a specified time period, typically 72 to 96 hours. The number of opposing submarines is generally known. Weapons are simulated by sound charges that can be carried in large numbers. These artificialities have, in past exercises, led to decisions to commit forces at a rate that would make them ineffective after 96 hours of operation. The exercise tactic of attacking every contact with a simulated weapon is not compatible with the number of actual weapons available, nor with the load capability of the delivery vehicle. In addition, the generation of simultaneous contacts in excess of the number of target submarines known to be assigned to the exercise, results in some contacts being suspect on a purely numerical basis. These factors bias tactical decisions and would not be present during wartime operations.

The effects of these exercise artificialities can be reduced or eliminated by realistic exercise limitations. However, too stringent limitations as to assignment of forces and expenditure of simulated weapons reduces the training and experience available to individual units participating in the limited exercise periods.

The critical parameter in the ASW commander's decision problems is the uncertainty associated with a given contact. Statistical data from exercises where valid reconstruction has been possible points up the degree of uncertainty associated with every contact. The effect of this uncertainty during exercise situations is somewhat obscured during the

exercise by the desire to work individual ASW units in every possible contact area. This is a spurious input factor to the required experience and "feel" of the decision maker.

The objective of any analysis of the ASW commander's decision problem is to identify a course of action that is logically consistent with the degree of contact uncertainty and the consequences associated with each available course of action. The analysis can reasonably be divided into two areas for discussion. The first area is that of contact uncertainty. The second area considers the consequances of each course of action.

Contact uncertainty, in the parlance of the statistician, is uncertainty due to the state of nature. A given contact is either a submarine or it is not. Here the two states of nature are submarine, or non-submarine.

The consequences of various courses of action can be analyzed by subjectively associating a value judgement or utility with each action. A given action will produce a desirability of result for each possible state of nature. This is more nearly a problem in prediction.

3. An Illustrative ASW Contact Situation.

The following hypothetical ASW contact situation will be used as a vehicle to discuss pertinent aspects of the decision problem.

Suppose that an ASW Hunter-Killer Task Group is escorting a mercantile convoy during its mid-ocean transit. The convoy is following an established convoy route. It is proceeding with a ten knot speed-of-advance. The ASW group has the responsibility of providing ASW protection for the convoy for a distance of 1,500 miles -- a six day transit period.

The hunter-killer group is composed of an ASW aircraft carrier with a deck loading of fixed-wing search aircraft (S-2), and sonar-dipping helicopters (SH-3). The carrier is escorted by seven destroyers.

The force is steaming in a typical disposition. Two S-2 search aircraft and four helicopters are maintained in a "ready" status on the carrier.

An S-2 search aircraft reports a disappearing radar contact and investigates the area with "Julie". A "Julie" echo confirmation is obtained. Two S-2 aircraft and four helicopters are launched from the carrier to assist in prosecuting the contact. Two destroyers in the vicinity of the contact are also sent to the contact area. One helicopter gains sonar contact; another helicopter classifies the contact as non-submarine. One of the alerted S-2 aircraft obtains a possible Julie fix; the other S-2 aircraft does not attempt localization. One destroyer gains sonar contact; the other destroyer classifies the contact as non-submarine.

One of the S-2 aircraft drops a weapon on the contact based upon

localization information. Weapon detonation is not observed. Two S-2 aircraft, two helicopters and one destroyer remain in the contact area to conduct close search and localization until the area of contact is well astern of the convoy.

During the time interval required for the convoy to clear the original area of contact, relief aircraft and helicopters are launched from the carrier to replace those previously on station in the contact area. While this contact is being actively prosecuted three additional contact incidents are generated by other units of the ASW force.

This hypothetical contact situation provides a typical scenario of actions and interactions of units within this ASW force. Imbedded within this contact situation are numerous tactical decision problems.

The initial report of a disappearing radar contact by the search aircraft was the result of a specific decision. The plane commander of the search aircraft decided that sufficient information was available to classify a particular sensor indication as a possible submarine.

The ASW group commander felt that the initial indication plus the confirming Julie echo was sufficient justification for investigation by additional forces. The commander's estimate of the validity of the contact, combined with an appreciation of the current tactical situation, dictated the number and type of additional units to be sent to the contact area.

The decision to attack the contact with a weapon was based on the aircraft commander's (or the contact area commander's) estimate of the validity of the contact just prior to weapon release.

The decision to maintain forces in the contact area until the

convoy was clear of the area was a result of the ASW commander's estimate of the threat, weighted by a subjective probability measure that the contact was in fact submarine.

This is a somewhat abbreviated description of the step by step analysis that would take place in this typical situation. Some of the actions and decisions cited are probably a result of standard operating procedures and policies rather than specific decisions resulting from conscious analysis at discrete periods in the tactical sequence. For instance, general guidelines are usually promulgated to indicate what combinations of sensor response will make an attack profitable.

Sonar has proven to be one of the best localization and classification equipments available. Development of the sonar-dipping helicopter provides the capability of putting sonar equipment into the contact area at an accelerated rate. The sonar equipment in a helicopter is generally less effective than destroyer sonar, but the helicopter arrives at the contact area much sooner than the destroyer. This development tends to dictate the types and numbers of units that the ASW commander will order to a contact area after sufficient weight is given to the degree of contact uncertainty and the current tactical situation.

Every decision made during a contact incident is ultimately the responsibility of the ASW commander. Some decisions are made by unit commanders within certain limitations previously specified by the ASW commander. The decision to launch a weapon is generally included in this area. For completeness, these decisions might be termed pre-planned decision rules. Each of these decision problems might profitably be analyzed and studied.

The major decisions, the ones which effectively control the overall actions of the force, are made at discrete intervals by the ASW commander. These are the decisions of primary interest here. The main emphasis will be upon the commander's decision to prosecute fully a given contact, or to "drop" it.

There is one aspect of this particular decision problem that appears unique. After a decision has been made to disregard a particular contact, it is often possible to hedge the decision. In the illustrative situation just cited, if the commander had decided to drop the contact, the decision might have been hedged by assigning, say, two aircraft to remain in the area until the initial contact location no longer posed a threat to the convoy.

4. The Statistical Decision Process.

An ASW commander faced with a contact decision problem has particular information available to him. He is aware of the overall tactical situation. He has been apprised of the technical aspects of the ASW equipment within his force, and the local environmental conditions.

The commander can solve the decision problem using an informal decision method. On the basis of known facts, his experience, judgement and intuitive feel, he can decide to drop the contact, or supply additional forces to fully prosecute the contact.

The other alternative is to systematically analyze each factor in the decision problem and apply relevant statistical decision rules to aid in the decision.

Most aspects of these two decision methods are quite similar. The statistical decision process is but a formal method of considering facts, assumptions, and objectives that bear on decisions under uncertainty.

The military reader will be aware of the step by step formalism in the commander's "Estimate of the Situation" set forth in the publication Joint Action Armed Forces. The major steps in the estimate are 1) Mission and its analysis; 2) Situation and courses of action; 3) Analysis of opposing courses of action; 4) Comparison of own courses of action; and 5) the Decision.

Steps in the statistical decision process follow closely this military planning outline. The emphasis in the statistical decision method is on the concept of "expected value" associated with a given action.

It will be well to discuss first the basic concepts of the decision process in terms of a simplified example, using the probabilists' classic urn.

Consider an urn containing 85 white balls. A handful of 30 black balls is tossed at the mouth of the urn. You are then offered the following proposition. An individual will reach into the urn and withdraw a single ball. Before the ball is withdrawn you are to guess whether the ball will be black or white. If you specify a black ball and a black ball is drawn you and the individual drawing the ball will win a total of $113; but if the ball is white you will both lose a total of $20. If you specify a white ball and a black ball is drawn your combined loss will be $255; but if the ball is white you gain a total of $45.

This information is displayed in the following payoff table.

EVENT	ACTION	
	A_w - Specify a white ball.	A_b - Specify a black ball.
E_b - Draw a black ball.	- $255	$113
E_w - Draw a white ball.	$45	- $20

When the black balls were tossed at the mouth of the urn, you estimate that 15 of the black balls dropped into the urn.

Your analysis of the situation might proceed in the following manner. From the information available and your observation of the toss of the handful of black balls, you feel that the urn now contains 100 balls; 85 white and 15 black. This would translate into probabilities of 0.15 for drawing a black ball and 0.85 for drawing a white ball.

With an intuitive approach you might feel that your chances for gain would be better if you specified a white ball, (i.e., took action

A_w). The "odds" appear to be in favor of a white ball being drawn.

Using the expected value approach, however, either action would be equally profitable. Putting the probabilities into the payoff table, we have:

EVENT	Probability of the event occurring.	ACTION	
		A_w - Specify a white ball.	A_b - Specify a black ball.
E_b - Draw a black ball.	0.15	- $255	$113
E_w - Draw a white ball.	0.85	$45	- $20

The fact that the event E_w has a relatively high probability of occurring weighs heavily in its favor. But event E_b, drawing a black ball, has some weight no matter how small. If the results of each action are weighed with respect to the proposed payoff and the probability of occurrence for each event, a "weighted average payoff", or expected payoff can be determined.

For action A_w the expected payoff is:

$$(0.15)(-\$255) + (0.85)(\$45) = \$\ 00$$

For action A_b the expected payoff is:

$$(0.15)(\$113) + (0.85)(-\$20) = \$\ 00$$

Using the statistical decision approach, the decision rule for maximizing gain (or minimizing loss) is to take the action having the higher expected return. In this example, either action could be specified with the same average return, zero dollars.

This decision rule is relative. Had the expected returns been equal to one cent and zero, deciding in favor of the action with a one

cent expected value would be somewhat marginal. The question is, how much "higher" should one's expected return be to justify deciding in its favor. This question must be considered within the context of the particular decision problem.

As a further illustration, suppose the same game of chance is proposed. But in addition, the individual drawing the ball from the urn is offered an opportunity to conduct a series of experiments. He is allowed to draw a ball from the urn, note its color, and then replace it. The experiment to be conducted a maximum of 100 times. You are also told that there is a minute difference in the surface texture of the white and black balls. In all other respects, save the color and texture, the balls are identical. When the experiments are completed a color is to be specified and a single ball is to be drawn, as before.

The payoff matrix will be the same as for the first example. The opportunity to conduct the experiments will cost $5, regardless of the outcome of the final gamble. This latter aspect is in accord with the somewhat realistic observation that information generally commands a price.

The individual drawing the balls from the urn is allowed to test the texture of one black and one white ball prior to conducting the preliminary sampling.

Because of the higher payoff for specifying a black ball and then drawing a black ball, the individual drawing the ball would like to select a black ball from the urn on the payoff draw. During the sampling experiment he attempts to select a black ball each time.

Suppose that in conducting the experiments 25 of the 100 balls drawn

are black. That is, in attempting to draw a black ball by testing the texture, the individual drawing the balls was successful 25 percent of the time.

This problem is somewhat less amenable to an intuitive decision due to the added complications of the experiments and the cost of experimenting.

For the statistical decision process, use is made of the following equation, due to Bayes, to weigh the information available.

$$P(E_i|S) = \frac{P(S|E_i)\, P(E_i)}{\sum_j P(S|E_j)\, P(E_j)}, \tag{1}$$

where $P(E_i|S)$ denotes the conditional or relative probability that event E_i will occur, given the hypothesis that S is known to have occurred.

An explicit form of equation (1), more pertinent to the example under discussion is:

$$P(E_b|S_b) = \frac{P(S_b|E_b)P(E_b)}{P(S_b|E_b)\, P(E_b) + P(S_b|E_w)\, P(E_w)} \tag{2}$$

In the example under discussion E_b and E_w are the events "draw a black ball" and "draw a white ball". The symbols S_b and S_w denote the hypothesis that the individual drawing the ball "says" that he is drawing or attempting to draw a black or a white ball.

$P(E_b)$ in equation (2) is the "a priori" probability of drawing a black ball. This is sometimes referred to as the actual or original probability. $P(E_b|S_b)$ is the "a posteriori" probability of drawing a black ball; often called the new or gained probability. The a posteriori probability is the new probability of drawing a black ball when the

information on the capability of the individual to draw a black ball is considered.

Using equation (2) to compute the compound probability of drawing a black ball:

$$P(E_b|S_b) = \frac{(0.25)(0.15)}{(0.25)(0.15) + (0.75)(0.85)} = 0.056 .$$

Considering the original estimate of the number of black and white balls in the urn, and the information gained from conducting the experiments the probability of drawing a black ball is now 0.056. The probability of drawing a white ball, under these conditions, is one minus this amount, or 0.944. Putting these probabilities into the payoff table, we have

EVENT	PROBA-BILITY	ACTION	
		A_w	A_b
E_b	0.056	-$260	$108
E_w	0.944	$40	- $25
Expected value:		$23.14	-$17.55

The individual payoffs reflect the five dollar information cost.

Using the statistical decision rule to choose the action with the higher expected payoff, action A_w, specify a white ball, is indicated.

It should be noted that the individual's attempt to draw a black ball produced a counter-effect on the decision. Even though he was attempting to draw a black ball, the experiments indicated that there was a positive conditional probability of 0.75 that he would draw a white ball. To disregard this information, with the idea that the

individual was not very adept in drawing black balls, would reduce the problem to the original example having equal expected payoffs.

If after reviewing the results of the experiments, the individual had decided to try to draw a white ball on the final gamble, the historical information gained from the experiments would be meaningless. The experimental conditions and the conditions under which the final draw would have been conducted would not have been the same.

Had the experiments resulted in a sample of 65 black balls in 100 draws the expected payoffs would have been - \$55 for action A_w and \$8.25 for action A_b. In this case action A_b, specify a black ball, would have been indicated.

These two examples have introduced two interpretations of probability and the concept of expected value.

Classically, probabilities have been interpreted in the relative frequency sense. For instance, the probability of observing "heads" on the toss of an average coin is 0.5. This means that if the coin is tossed over and over again, the number of heads observed will tend to equal the number of tails observed. It does not mean that if the coin is tossed ten times that five heads will be observed.

Using a decision rule favoring the higher expected value, in this sense of a "long-run average", would be valid only for events that are to be repeated over and over again. To use the long-run average based on the relative frequency interpretation of probability to specify the outcome of a single toss of the coin would have no meaning.

Personal or subjective probabilities are not assertions of relative frequencies. They are instead a measure of an individual's subjective

feeling as to the probable outcome of a particular event. Subjective probabilities are often the same as relative frequency probabilities, as is generally the case for the toss of a coin. This is an indication that the only information available is the historical results gained from tossing average coins. But if additional information were available regarding a particular coin, that it was bent, or that it was peculiarly weighted, then relative frequency would be only one factor to consider in specifying a personal probability for the toss of this particular coin.

Using expected value as a guide to decision on the basis of personal probabilities is not dependent on the long-run average and the repetition of an event. It can logically be used as the basis for a single event decision.

5. The Statistical Decision Process Applied to the ASW Problem.

The statistical decision concepts of the previous sections can be applied to the anti-submarine warfare contact situation. Pertinent factors in the hypothetical contact situation set forth in Section 3 can be analyzed in a manner similar to that used in the ball and urn example.

Somewhat realistically, it will be assumed that the ASW commander has access to the following information:

a. Intelligence estimates indicate that from 7 to 15 of the enemy's fleet of submarines are probably assigned to the 1,500 mile segment of the convoy route under discussion.

b. Records of previous exercises and convoy operations show that each airborne search unit will generate an average of 15 contact incidents for each 10,000 square miles searched.

c. There is a statistical measure of uncertainty for initial detection and contact confirmation incidents that can be associated with each ASW vehicle/sensor type. The uncertainty measures are in the form of relative frequency probabilities obtained from reconstructed ASW exercises.

Faced with a typical contact decision problem, the ASW commander must choose between two possible courses of action:

A_n - "Drop the contact"; no further attempt is made to localize or attack the contact.

A_s - "Prosecute the contact"; provide additional forces to attempt localization and kill.

There are a large number of possible actions available to the ASW commander, most of which are not relevant to the problem at hand.

Although there are more than two feasible and relevant courses of action available, the present discussion will be limited to the two actions stated above.

In terms of the previous discussion of the statistical decision process, there are two possible events or outcomes; two possible "states of nature".

E_n - The contact <u>is not</u> a submarine.

E_s - The contact <u>is</u> a submarine.

The third factor needed to construct the payoff matrix is the payoffs themselves. Monetary payoffs of the kind used in the ball and urn example are not entirely meaningful here. The concept of utility or utility-value is broader and can include the monetary aspect.

The ASW commander makes a value judgement as to the relative utility of taking a particular action under the assumption that the contact is or is not a submarine. These value judgements are transformed into numerical values which are intended to describe the commander's subjective ideas as to the relative value of specific outcomes. Inherent in this concept of utility-value is the commander's estimate of the consequences of disregarding a contact that is in fact a submarine, of prosecuting a contact that is not a submarine, etc. These utility-values are not constant, they can and will change with changes in the tactical situation and the mission of the ASW force.

For the hypothetical contact situation previously cited, the numbers representing an assumed set of such value judgements are listed in the payoff table below.

EVENT	ACTION	
	A_n - Drop the contact.	A_s - Prosecute the contact.
E_n - Contact is not a submarine.	50	-15
E_s - Contact is a submarine.	-250	110

The utility values in the payoff matrix are relative with respect to one another. The numbers could all be made positive by adding 250 to each one, and their relative ordering would remain unchanged. In addition, such an adjustment would not affect the final indication in the statistical decision process. The use of positive and negative values is to provide a sense of physical gain or loss.

The final factor to be considered is the degree of contact uncertainty. For this example suppose that the following relative frequency probabilities are available from reconstructed exercises.

INITIAL DETECTION INCIDENTS

Vehicle/Sensor	Valid Detections	False Detections	Percent Valid
S-2			
Radar	12	48	20
.	.	.	.
.	.	.	.
DD			
Sonar	40	93	30
.	.	.	.
.	.	.	.

CONTACT CONFIRMATION INCIDENTS

True State of Nature	Submarine			Non-Submarine		
Vehicle/Sensor Classification	Sub.	Non-Sub.	% Corr.	Non-Sub.	Sub.	% Corr.
S-2						
Julie	24	56	30	80	50	40
.	.	.	.	.	.	.
.	.	.	.	.	.	.
SH-3						
Sonar	30	45	40	56	24	70
.	.	.	.	.	.	.
.	.	.	.	.	.	.
DD						
Sonar	42	28	60	60	15	80
.	.	.	.	.	.	.
.	.	.	.	.	.	.

The above data are hypothetical, developed for illustrative purposes only. The data are categorized by initial detection and contact confirmation to provide for the two modes of crew operation. In a confirmation situation the sensor operator, and in fact the entire crew, is in an "alerted" condition.

These statistics provide the conditional probabilities referred to in the urn problem. For instance, the helicopter sonar confirmation data provide the conditional probability $P(HS_s | E_s)$, the probability that the helicopter sonar detection system will indicate "submarine" when a submarine is actually present. The data give this particular probability as 0.40.

If Bayes' rule is to be used a final statistic, not obtainable from the above data, is required. As in the urn problem, the a priori or initial probability must be known or approximated. Equate the initial number of black and white balls in the urn to the number of "submarines"

and "non-submarines" in the particular ocean area of interest. The required probabilities are $P(E_s)$, the ratio of the number of submarines present to the total number of submarines plus non-submarines present; and $P(E_n)$, the ratio of the number of non-submarines present to the same total of submarines plus non-submarines. The non-submarines are the phenomenon and objects in the ocean that produce submarine-like indications in search sensors.

In the case of airborne systems, suppose that each aircraft generates an average of 15 contact incidents for each 10,000 square miles of search, irrespective of the density of actual submarines. This can be used as an approximation to the total number of submarine and non-submarine contacts in each 10,000 square miles of search area, for each aircraft.

For this example, assuming a search area 50 miles on either side of the convoy track, search operations will cover 150,000 square miles. This translates into approximately 225 non-submarine and submarine contacts in the search area available to each search aircraft.

The intelligence information assumed in this example places 7 to 15 enemy submarines in the same 150,000 square miles of ocean. Suppose that past exercise operations indicate that aircraft detection systems will detect 65 percent of the actual number of submarines in the exercise search area. Using this approximation and the maximum number of enemy submarines expected to be in the search area, the number of submarines in the total population of airborne contact incidents is approximated at 9.7 .

This line of reasoning produces the following a priori probabilities

for aircraft search systems.

$P(E_s) = 0.043 \qquad\qquad P(E_n) = 0.957$

The ASW commander is aware of the source and sample size of the statistical information regarding initial detection and contact confirmation. In this example suppose that the majority of exercises from which these data were obtained were conducted in areas of "good" sonar conditions and sea states of 2 to 3. Current sonar conditions are "good" and the sea state is 0 to 1. This information is used to modify the relative frequency probabilities provided by the data.

For instance, the conditional probability for initial S-2/radar detection is indicated as $P(S_s|E_s) = 0.20$. The commander assigns a subjective probability of 0.25 to this detection, based primarily on the current reduced sea state. The others are modified in a similar manner, where applicable. To summarize the relevant probabilities:

The a priori probabilities:

$P(\text{Submarine}) = P(E_s) = 0.043$

$P(\text{Non-submarine}) = P(E_n) = 0.957$

Initial detection probabilities:

S-2 radar: $P(SR_s|E_s) = 0.25$

$P(SR_s|E_n) = 0.75$

Confirmation probabilities:

S-2 Julie: (contact) $P(SJ_s|E_s) = 0.30$

$P(SJ_s|E_n) = 0.60$

Helicopter sonar: (contact) $P(HS_s|E_s) = 0.40$

$P(HS_s|E_n) = 0.30$

Helicopter sonar: (no contact) $P(HS_n|E_s) = 0.60$

$P(HS_n|E_n) = 0.70$

Destroyer sonar: (contact)	$P(DS_s \mid E_s) = 0.60$
	$P(DS_s \mid E_n) = 0.20$
Destroyer sonar: (no contact)	$P(DS_n \mid E_s) = 0.40$
	$P(DS_n \mid E_n) = 0.80$

The a posteriori probability to be determined in this example is the probability that the contact is a submarine given the conditions that the S-2 radar classified the contact as submarine; S-2 Julie classified it as a submarine; one helicopter classified it as a submarine; one helicopter classified the contact as non-submarine; one destroyer gained sonar contact; and one destroyer classified it as non-submarine. In symbols, the desired probability is $P(E_s \mid DS_s\ DS_n\ HS_n\ HS_s\ SJ_s\ SR_s)$.

If the initial detection and subsequent confirmations are considered step by step, one action at a time, the a posteriori probability determined by one action becomes the a priori probability for the next action to be considered.

For the initial radar detection, using Bayes' formula,

$$P(E_s \mid SR_s) = \frac{P(SR_s \mid E_s)\ P(E_s)}{P(SR_s \mid E_s)\ P(E_s) + P(SR_s \mid E_n)\ P(E_n)} .$$

The a posteriori probability that the contact is a submarine, using the information gained from the initial detection, is

$$P(E_s \mid SR_s) = \frac{(0.25)\ (0.043)}{(0.25)(0.043) + (0.75)(0.957)} = 0.015 .$$

Next, the information available from the S-2 Julie confirmation is considered. The a posteriori probability for the initial detection, $P(E_s \mid SR_s)$, is used as the a priori probability, $P(E_s)_1$, for the next calculation.

$$P(E_s|SJ_s\ SR_s) = \frac{P(SJ_s|E_s)\ P(E_s)_i}{P(SJ_s|E_s)\ P(E_s)_i + P(SJ_s|E_n)\ P(E_n)_i}$$

So that,

$$P(E_s|SJ_s\ SR_s) = \frac{(0.30)(0.015)}{(0.30)(0.15) + (0.60)(0.985)} = 0.0075\ .$$

Continuing with each successive information factor, we have

$P(E_s|HS_s\ SJ_s\ SR_s) = 0.0099,$

$P(E_s|HS_n\ HS_s\ SJ_s\ SR_s) = 0.0085,$

$P(E_s|DS_n\ HS_n\ HS_s\ SJ_s\ SR_s) = 0.0043,$

$P(E_s|DS_s\ DS_n\ HS_n\ HS_s\ SJ_s\ SR_s) = 0.013.$

The final a posteriori probability that the contact is a submarine is 0.013. The probability that the contact is not a submarine is one minus this number, or 0.987. Putting these values into the payoff matrix and calculating the expected value,

EVENT	PROBA-BILITY	ACTION	
		A_n	A_s
E_n	0.987	50	-15
E_s	0.013	-250	110
Expected value		46.1	-13.4

The decision rule which favors the action with the higher expected payoff would indicate the selection of action A_n, drop the contact. An additional criterion is provided when the expected values are considered within a relative frame of reference.

If each of the vehicle/sensors had classified the contact as "submarine", the final a posteriori probabilities would have been 0.109 for

event E_s and 0.891 for event E_n. The expected values would have been -17.3 for action A_n and -1.4 for action A_s.

If each classification attempt had resulted in a "non-submarine" classification, the probabilities would have been 0.005 for event E_s and 0.995 for event E_n. The associated expected values for these probabilities are 48.5 for action A_n and -14.5 for action A_s.

Intuitively, these two results appear to mark the extreme values that could be expected from this particular interaction. However, a wider range of values could occur. This situation is similar to the one discussed in the urn problem, where it was profitable to "bet against the individual selecting the ball."

For the submarine contact situation, one limiting set of values would occur if the S-2/Julie sensor classified the contact as non-submarine and the other units classified it as a submarine. This would result in probabilities of 0.30 for event E_s and 0.70 for event E_n. The expected values would then be -40 for action A_n and 22.5 for action A_s.

The other limit would be defined for a S-2/Julie classification of submarine, with all other units classifying the contact as non-submarine. The probabilities would then be 0.0014 for event E_s and 0.9986 for event E_n, with expected values of 49.6 for action A_n and -49.6 for action A_s.

Comparison of the calculated expected values with the extreme values which could be anticipated can best be done by referring to the absolute differences in expected values. This difference in expected values can be considered as a measure of "risk" when referred to the action with the lower expected value.

For the calculated values of 46.1 and -13.4 this absolute difference

is 59.5 when referred to action A_s. In this case, to select action A_s instead of A_n the decision-maker is risking an amount 59.5. A zero risk value would occur when the expected values for each action were zero. The decision-maker would then be indifferent as to the choice of a particular action.

Calculating the risk values for the limiting conditions, we have

99.2	Maximum value
(63.0)	(All vehicle/sensors classify contact as non-submarine)
59.5	<u>Actual classification information from vehicle/sensors</u>
0.0	Point of indifference
(-15.9)	(All vehicle/sensors classify contact as submarine)
-62.5	Minimum value

This method of comparing expected values provides a measure of relative magnitudes. It also permits comparisons with other contact incidents.

Where physical considerations limit the number of contacts that can be prosecuted simultaneously, a comparison of risk values will indicate the contacts which can be prosecuted most profitably.

6. Sensitivity Analysis.

The a posteriori probabilities determined in the previous analysis were calculated to the third and sometimes the fourth significant figure. It will be said that it is unrealistic to generate numbers which infer four place accuracy when the input probabilities are subjectively determined. But it can be argued that the input probabilities can be made as accurate as one desires them to be. If a probability of 0.90 is subjectively assigned for the occurrence of a particular event, an extension of this same type of subjective evaluation will permit one to opinionize a probability of 0.9032 with similar subjectivity. Probability in this sense is but ordered opinion, and the ordering can be as definitive as is required.

In this particular application rounding-off probabilities to the first decimal place would fatally degrade the sensitivity of the individual classification inputs. There appears to be sufficient sensitivity in this type of analysis when input probabilities of two or three significant figures are used.

It will be noted that the sample size is quite small for initial contact and confirmation data. A data base of this size generally is not considered sufficient to provide statistical validity. This factor must be weighed by the decision-maker in making the final comparison of risk-values. A change in one of the recorded incidents creates a sizeable change in the conditional probabilities.

In the "submarine" incidents for DD sonar confirmation, the total sample size is 70 -- 42 correct classifications and 28 incorrect classifications. These figures provide a probability $P(DS_S | E_S) = 0.60$.

A change in one incident, say to 43 correct and 27 incorrect classifications, would change this probability to 0.615.

The hypothetical sample sizes used in this illustrative problem are intended to emphasize the relatively limited data base that is currently available. This shortcoming can be corrected.

Bayes' theorem of conditional probability is far from being a new idea in statistics. However, opposition to its use has been widespread. This reluctance has stemmed from the difficulty of obtaining the required a priori probabilities. With the historic or relative frequency view of probability, the prior probabilities are required to be prohibitively accurate. Introduction of subjective probability, the view that probabilities are a measure of one's personal opinion, has effectively removed this obstacle -- provided that this definition for the measurement of uncertainty is accepted.

This is not to say that these probabilities are arbitrary. The a priori probability $P(E_s)$ is really a conditional probability. It is the probability of the contact being submarine, based on all of the available information about enemy submarines in the area, prior to the time the initial detection is made.

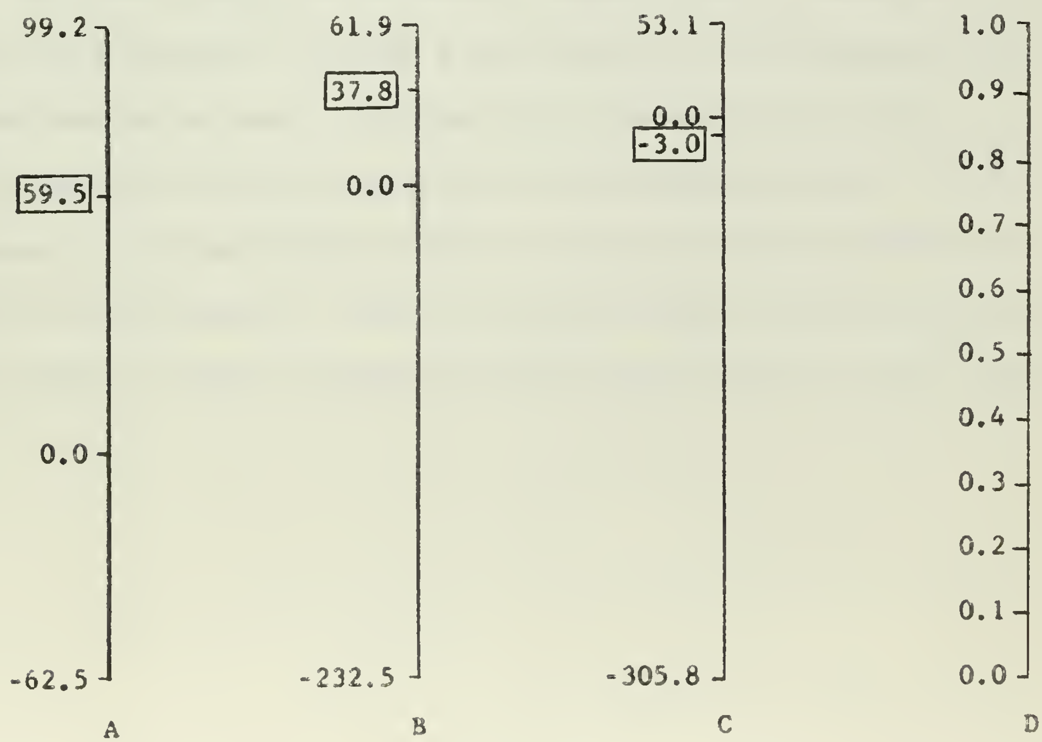
A major advantage in using Bayes' theorem is that two logical but different a priori probabilities will converge toward the same a posteriori probability with successive applications of Bayes' formula. The a priori information becomes overwhelmed by the successive weighing of additional information.

Suppose that the initial probabilities in the submarine example had been 0.20 for the contact being a submarine and 0.80 for the probability

of a non-submarine. These probabilities in the illustrative analysis were 0.043 and 0.957. With this set of different prior probabilities the final a posteriori probability would have been 0.065 for the contact being a submarine. This would have produced a risk value of 37.8. The minimum and maximum risk then would have been -232.5 and 61.9.

Experienced ASW commanders would not vary radically in determining the values for initial probabilities. In the highly unlikely circumstance that the a priori probabilities were taken as 0.50 and 0.50, an extreme limit would be reached. This would be the condition of equally likely occurrence. This situation is sometimes referred to as the condition of equal distribution of ignorance, or the condition of insufficient reason. Equally likely events are those in which, after exhaustive examination of the available information influencing the event, one is led to assume that no particular event will occur in preference to the other.

The following graph compares the two conditions just discussed and the results of the original analysis.



Line A is the resulting risk-values for the original analysis with a priori probabilities of 0.043 and 0.957. Prior probabilities of 0.20 and 0.80 determined the results for B. The equally likely case is represented by C. The last line, D, is a scale for the intervals of risk-values involved. The numbers in boxes are those values determined by the initial detection and subsequent confirmations. The extreme values for each case represent the maximum and minimum values that could have been obtained for the particular vehicle/sensors involved in the illustrative incident.

It will be noted that the boxed value is in the upper one-fourth of the risk interval in each case. As the prior probability for the submarine being present increases, the point of indifference, 0.0, moves toward the upper limit for the particular interval. This is as expected. The amount of movement is greater for small values of $P(E_s)$.

The prior conditions for graph A assume that 15 submarines are in the area of search. For graph B this equates to an assumption of 112 submarines in the area. For graph C this number would be 285.

A realistic view of the number of enemy submarines that would be expected in a given search area will limit the a priori probabilities to those less than 0.20. This will tend to reduce the sensitiveness in the analysis created by large variations in the initial probabilities.

7. Information Decisions.

In the previous analysis of the submarine contact incident it was assumed that the ASW commander was faced with a single contact decision problem at a specific point during the incident. This will not be the case in general.

As the tactical situation progresses each contact gained by a sensor, and each period where contact is not made on the potential target, creates an interim decision point. In the illustrative situation the arrival of the helicopters in the contact area prior to the arrival of the destroyers, is an example of one such decision point.

A decision analysis made at this point in the incident would have provided a risk-value of 61.4, with maximum and minimum values of 62.6 and 45.9. The indication here would have been for a decision to drop the contact at this point.

However, if no other contact incidents were in progress, or if it were possible to pursue this contact with a minimum loss of vehicle utilization, it might be profitable to delay an action decision until additional information were obtained.

If additional incidents had been in progress at the time of the initial contact referred to in the illustration, a comparative analysis of risk-values for each incident would have been in order. Such a comparison provides a method of evaluating the cost of obtaining additional information for any one contact incident.

This type of analysis permits the ASW commander to utilize his available forces most effectively.

8. Tactical Use of the Bayesian Decision Process.

The sea-going officer will view the foregoing analytical procedures with some trepidation. The mathematical methods are somewhat cumbersome and involved, or at best impractical for tactical use.

It might be suggested that a computer program be developed for the shipboard Navy Tactical Data System which would accomplish the required mathematical manipulations and display the results. This would be in keeping with the command and control functions for which this type computer system was developed. However, the present meagerness of statistical data, coupled with the non-availability of NTDS type computers for ASW use, postpones a payoff for such a proposal to the distant future.

A relatively simple method which would accomplish the same task could be made available to ASW forces at the present time. The available statistical data for initial detection and contact confirmation could be reduced to graphic form. This would provide a means for determining the necessary contact probabilities. Elementary nomographs for computing expected values also could be made available.

The graphs at the end of this section are examples of the type of graphs suggested. These samples were developed from the hypothetical data used in the previous submarine contact illustration.

Figure 1 would be used for the initial contact. Figures 2 through 7 would provide sequential probabilities as the interaction developed. Figures 8 through 11 are examples of nomographs for computing expected values.

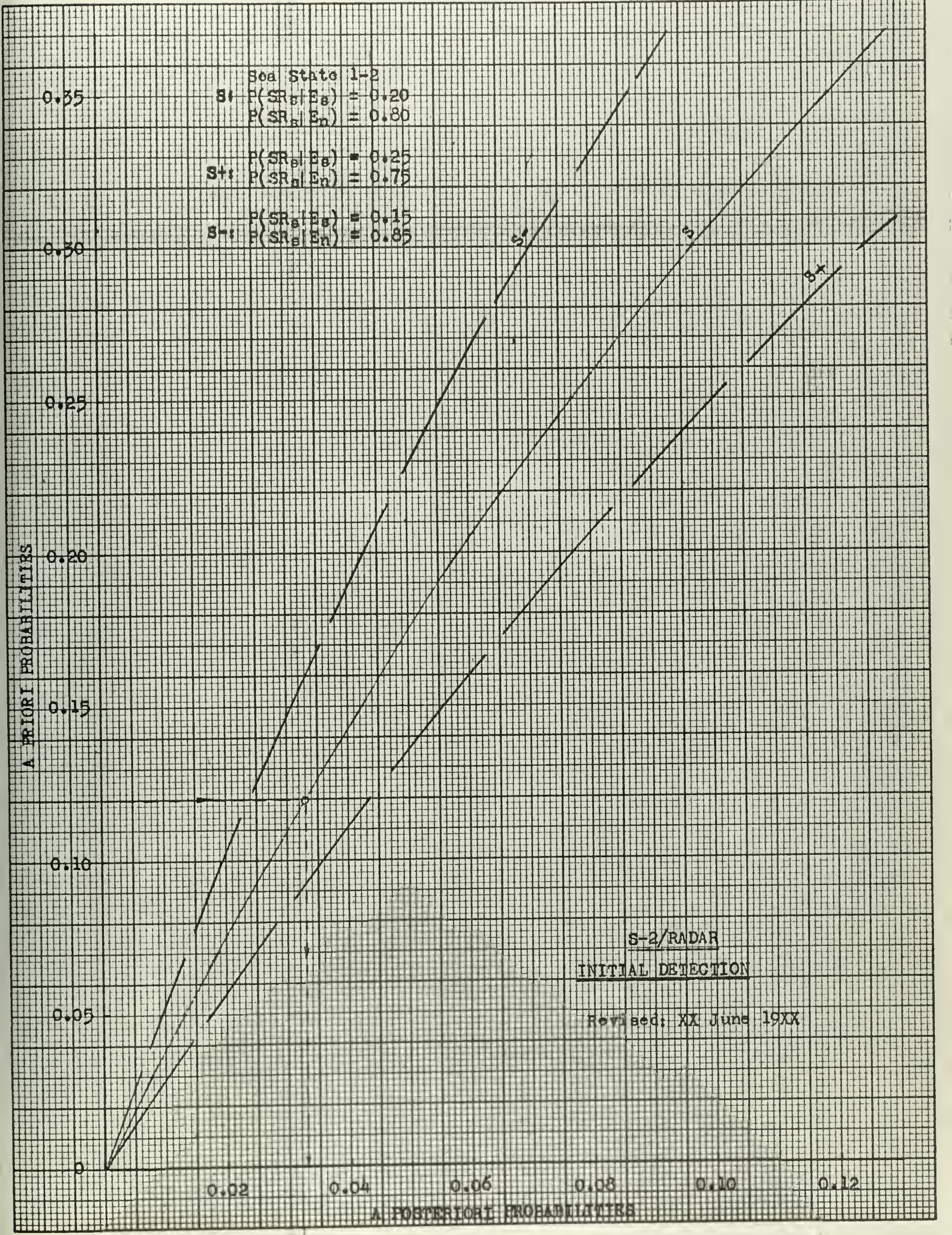
The probability graphs contain three curves. The solid line

represents statistical results for the exercise conditions stated in the chart. The broken lines, labeled (+) and (-), provide a method of increasing or decreasing the solid-line probabilities by a factor of 0.05. This permits the decision-maker to modify the historical data on the basis of current environmental conditions.

The a posteriori probability obtained from one graph is used as the a priori probability for evaluating the information factor next in sequence.

This type of graphical presentation, based on stored exercise statistics, can be produced with the use of particular auxiliary equipments presently available at major computer installations.

FIGURE 1



Sea State 1-2
S: P(SRs|Es) = 0.20
P(SRs|En) = 0.80
S+: P(SRs|Es) = 0.25
P(SRs|En) = 0.75
S-: P(SRs|Es) = 0.15
P(SRs|En) = 0.85
S-
S
S+
A PRIORI PROBABILITIES
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0.02
0.04
0.06
0.08
0.10
0.12
A POSTERIORI PROBABILITIES
S-2/RADAR
INITIAL DETECTION
Revised: XX June 19XX

FIGURE 2

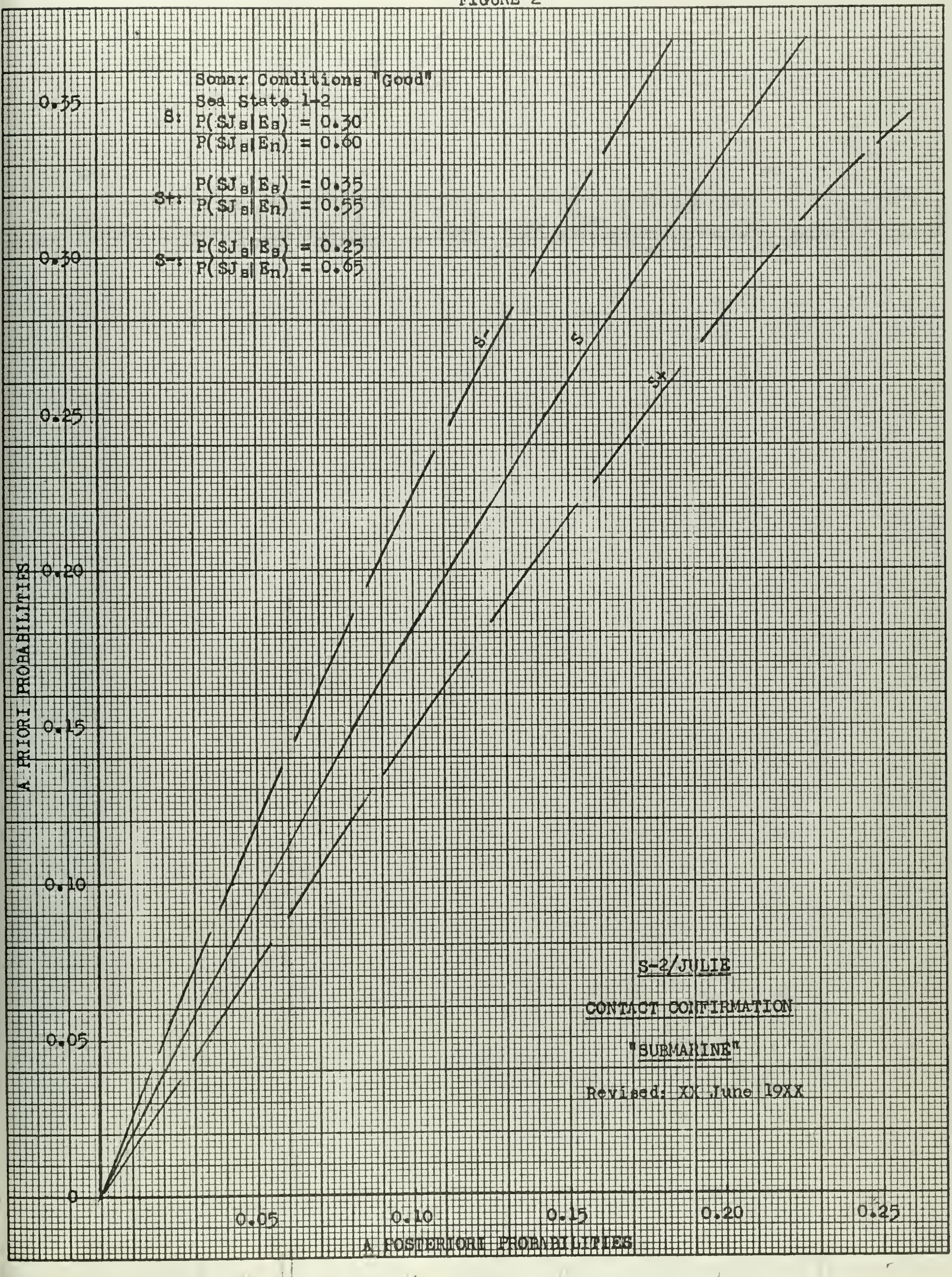
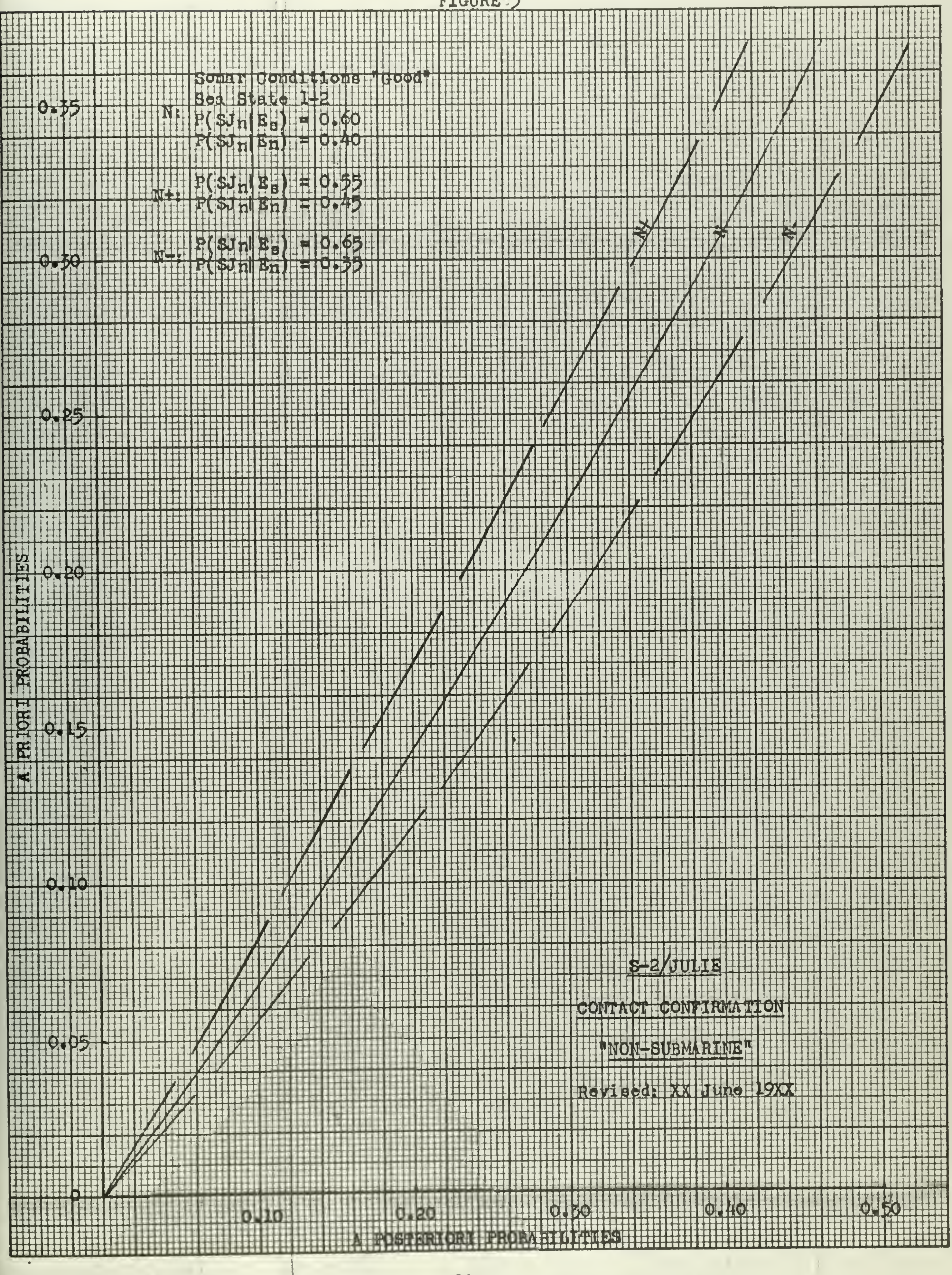

Sonar Conditions "Good"
Sea State 1-2
S: P(SJs|Es) = 0.30
P(SJs|En) = 0.60
S+: P(SJs|Es) = 0.35
P(SJs|En) = 0.55
S-: P(SJs|Es) = 0.25
P(SJs|En) = 0.65
S-
S
S+
S-2/JULIE
CONTACT CONFIRMATION
"SUBMARINE"
Revised: XX June 19XX
0
0.05
0.10
0.15
0.20
0.25
0.30
0.35
A PRIORI PROBABILITIES
A POSTERIORI PROBABILITIES

FIGURE 3



Sonar Conditions "Good"
Sea State 1-2
N: P(SJn|Es) = 0.60
P(SJn|En) = 0.40
N+: P(SJn|Es) = 0.55
P(SJn|En) = 0.45
N-: P(SJn|Es) = 0.65
P(SJn|En) = 0.35
N+
N
N-
A PRIORI PROBABILITIES
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0.10
0.20
0.30
0.40
0.50
A POSTERIORI PROBABILITIES
S-2/JULIE
CONTACT CONFIRMATION
"NON-SUBMARINE"
Revised: XX June 19XX

FIGURE 4

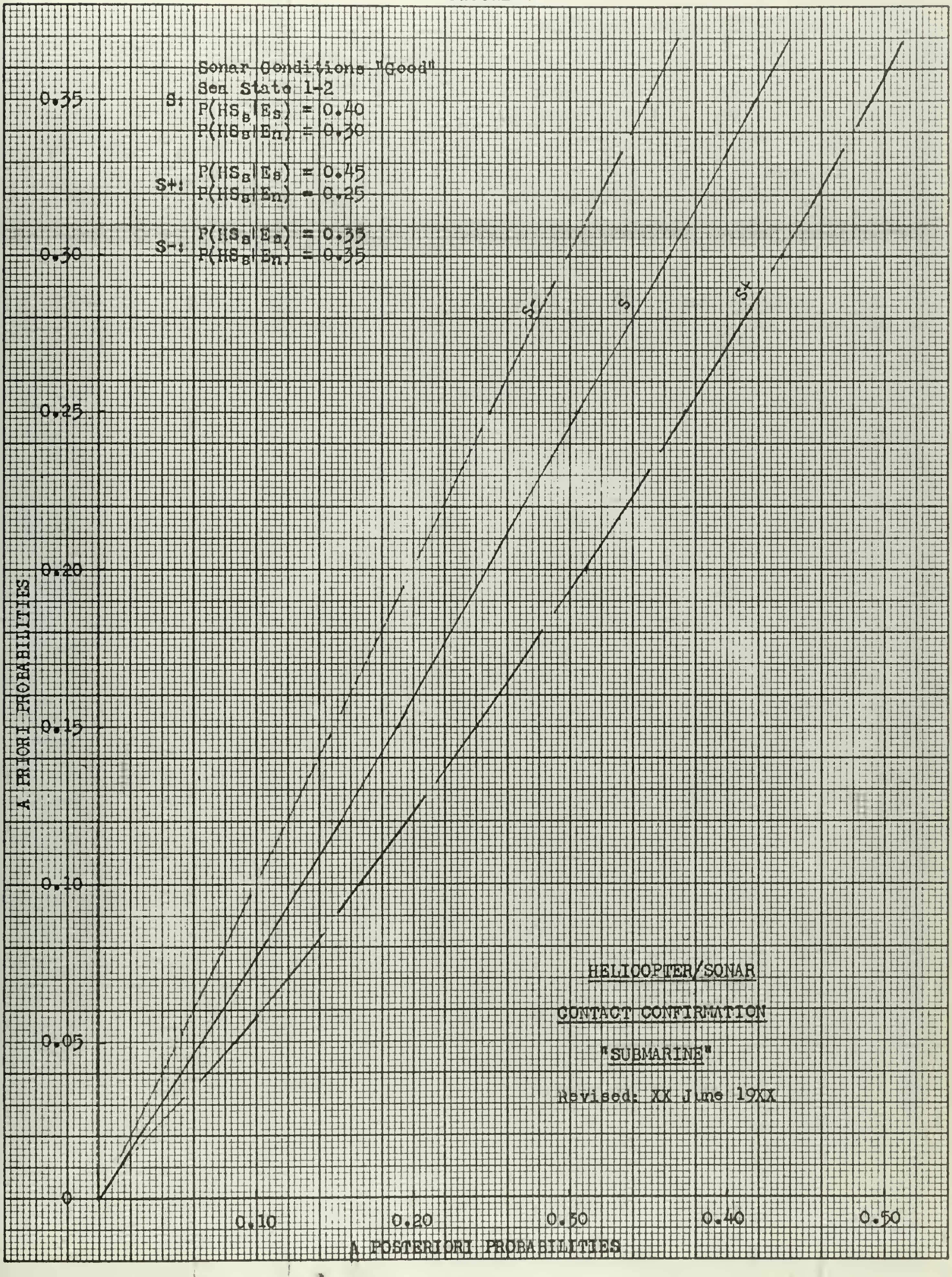

Sonar Conditions "Good"
Sea State 1-2
S: P(HS$_s$|Es) = 0.40
P(HS$_s$|En) = 0.30
S+: P(HS$_s$|Es) = 0.45
P(HS$_s$|En) = 0.25
S-: P(HS$_s$|Es) = 0.35
P(HS$_s$|En) = 0.35
S-
S
S+
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0.10
0.20
0.30
0.40
0.50
A PRIORI PROBABILITIES
A POSTERIORI PROBABILITIES
HELICOPTER/SONAR
CONTACT CONFIRMATION
"SUBMARINE"
Revised: XX June 19XX

FIGURE 5

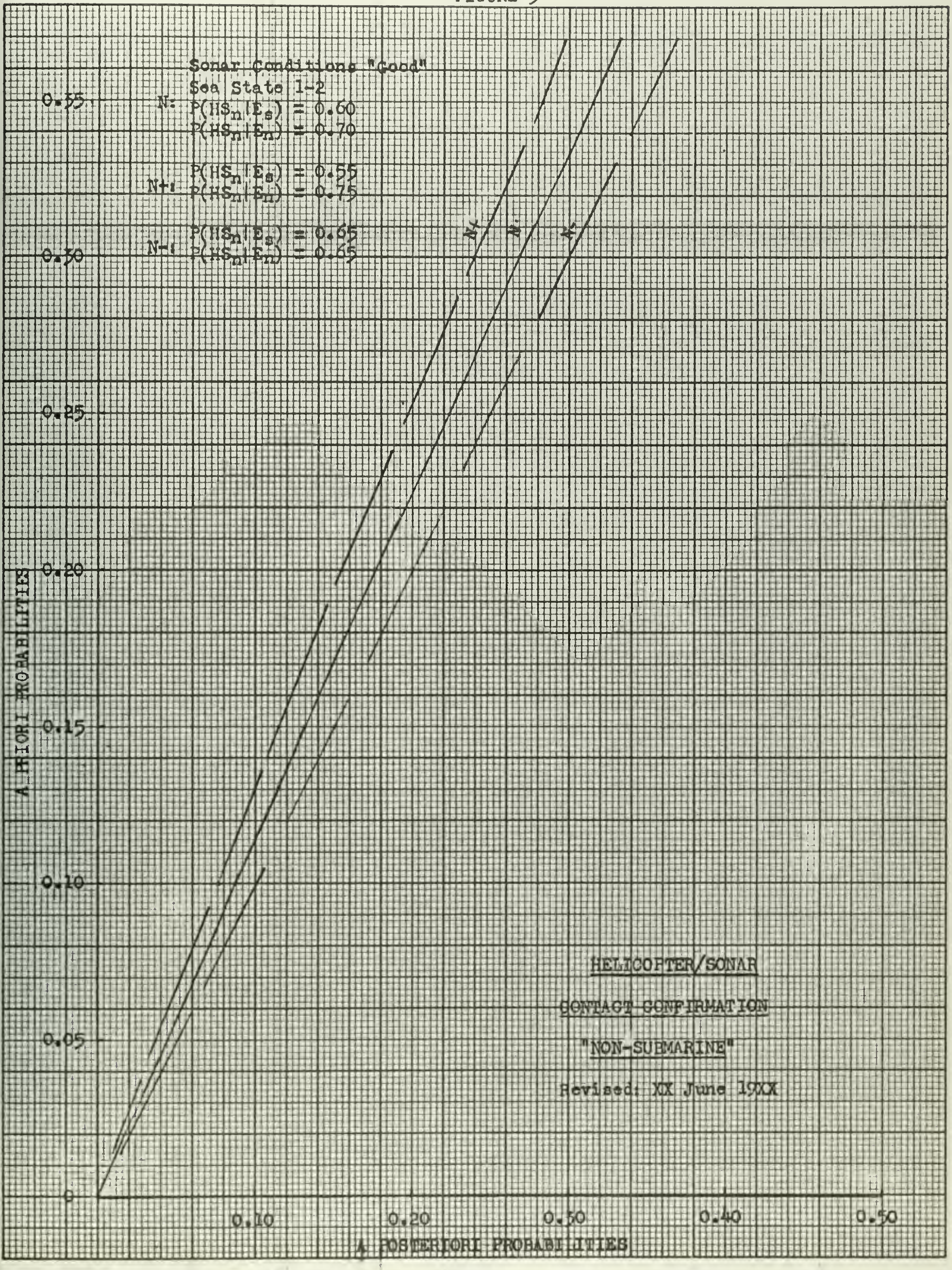

Sonar Conditions "Good"
Sea State 1-2
N: P(HS$_n$|E$_s$) = 0.60
P(HS$_n$|E$_n$) = 0.70
N+1 P(HS$_n$|E$_s$) = 0.55
P(HS$_n$|E$_n$) = 0.75
N-1 P(HS$_n$|E$_s$) = 0.65
P(HS$_n$|E$_n$) = 0.65
HELICOPTER/SONAR
CONTACT CONFIRMATION
"NON-SUBMARINE"
Revised: XX June 19XX
A PRIORI PROBABILITIES
A POSTERIORI PROBABILITIES
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0.10
0.20
0.30
0.40
0.50

FIGURE 6

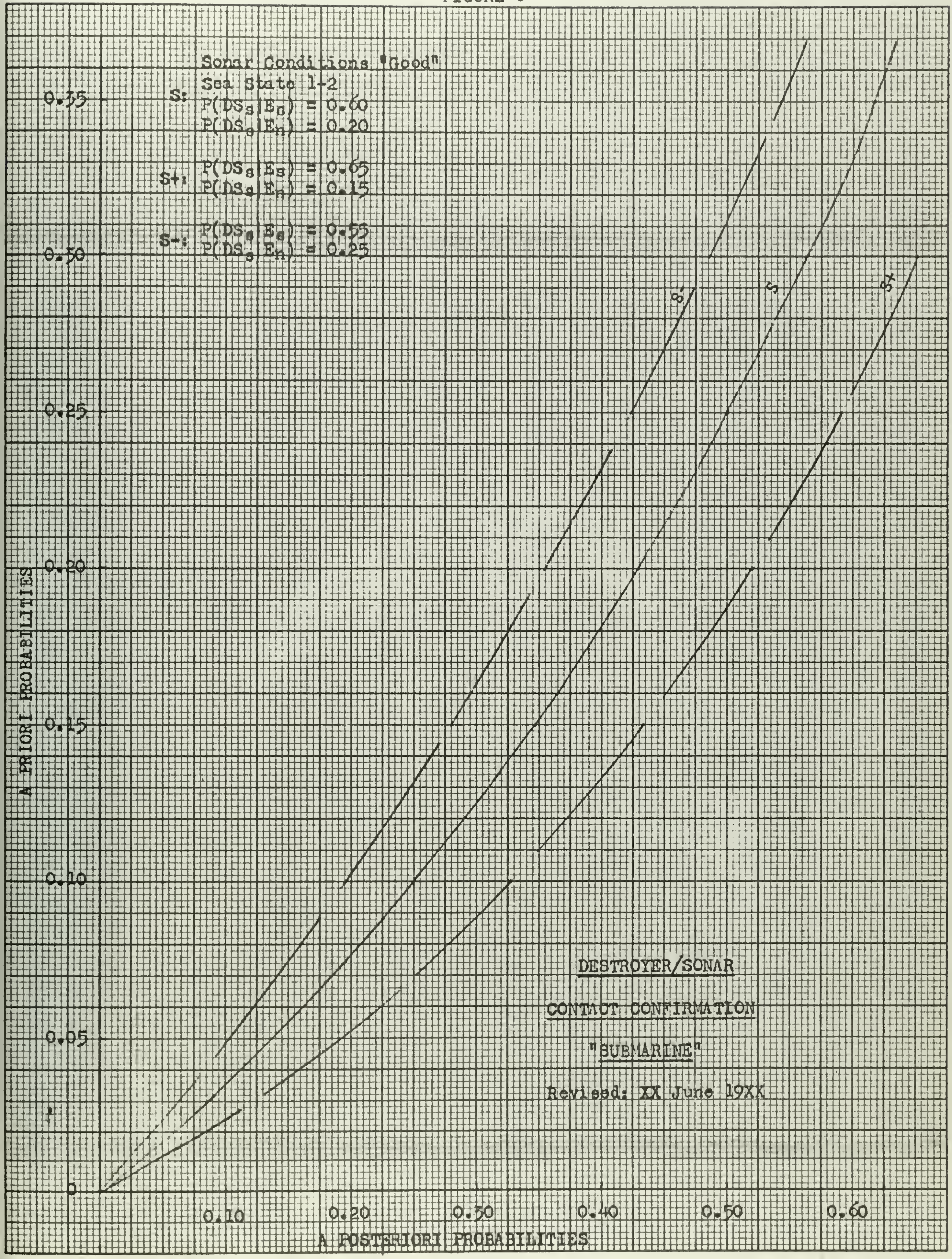
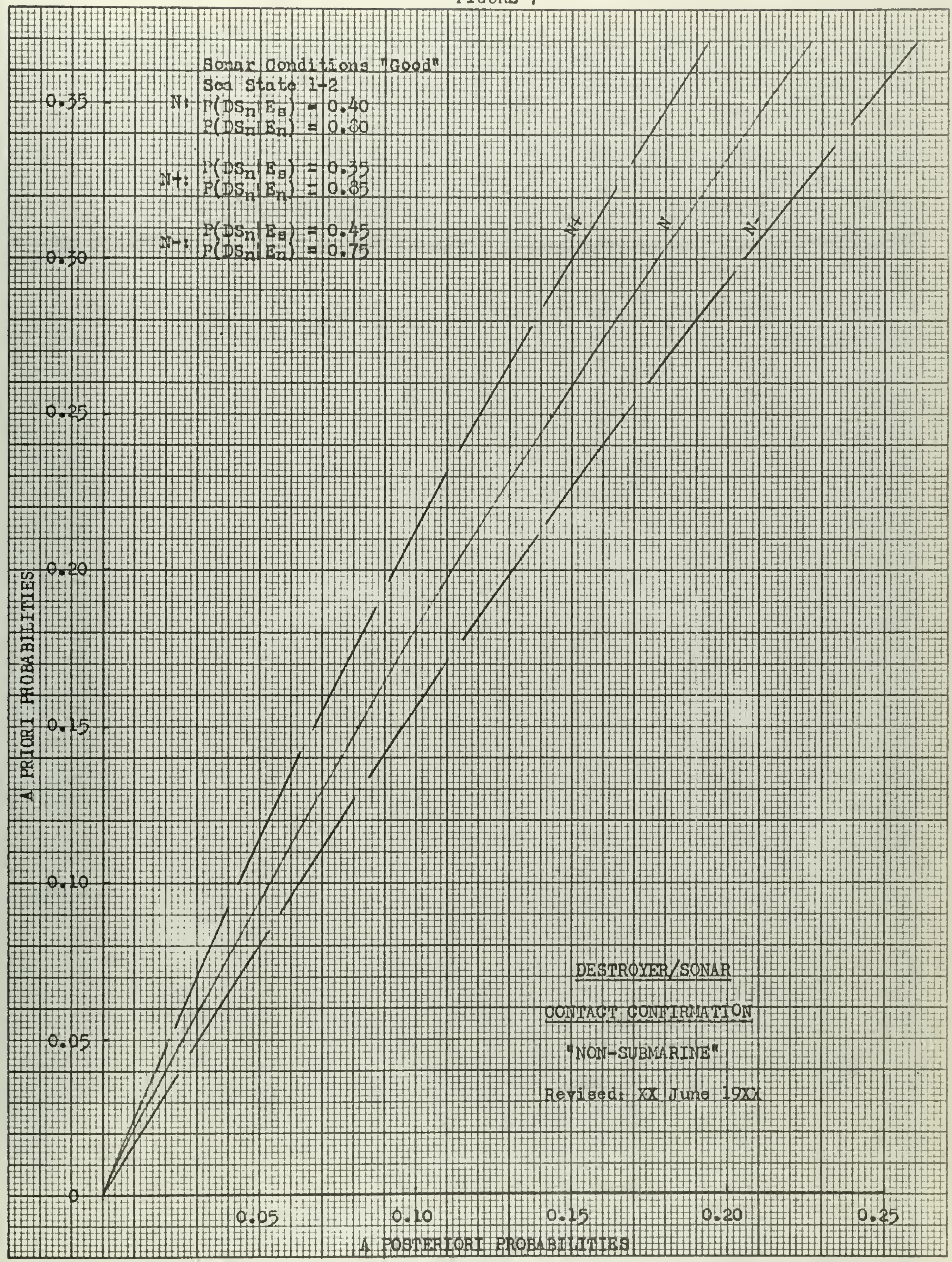

Sonar Conditions "Good"
Sea State 1-2
S: P(DSs|Es) = 0.60
P(DSs|En) = 0.20
S+: P(DSs|Es) = 0.65
P(DSs|En) = 0.15
S-: P(DSs|Es) = 0.55
P(DSs|En) = 0.25
S-
S
S+
DESTROYER/SONAR
CONTACT CONFIRMATION
"SUBMARINE"
Revised: XX June 19XX
A PRIORI PROBABILITIES
0
0.05
0.10
0.15
0.20
0.25
0.30
0.35
A POSTERIORI PROBABILITIES
0.10
0.20
0.30
0.40
0.50
0.60

FIGURE 7



Sonar Conditions "Good"
Sea State 1-2
N: P(DSn|Es) = 0.40
P(DSn|En) = 0.80
N+: P(DSn|Es) = 0.35
P(DSn|En) = 0.85
N-: P(DSn|Es) = 0.45
P(DSn|En) = 0.75
N+
N
N-
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0.05
0.10
0.15
0.20
0.25
A PRIORI PROBABILITIES
A POSTERIORI PROBABILITIES
DESTROYER/SONAR
CONTACT CONFIRMATION
"NON-SUBMARINE"
Revised: XX June 19XX

FIGURE 8

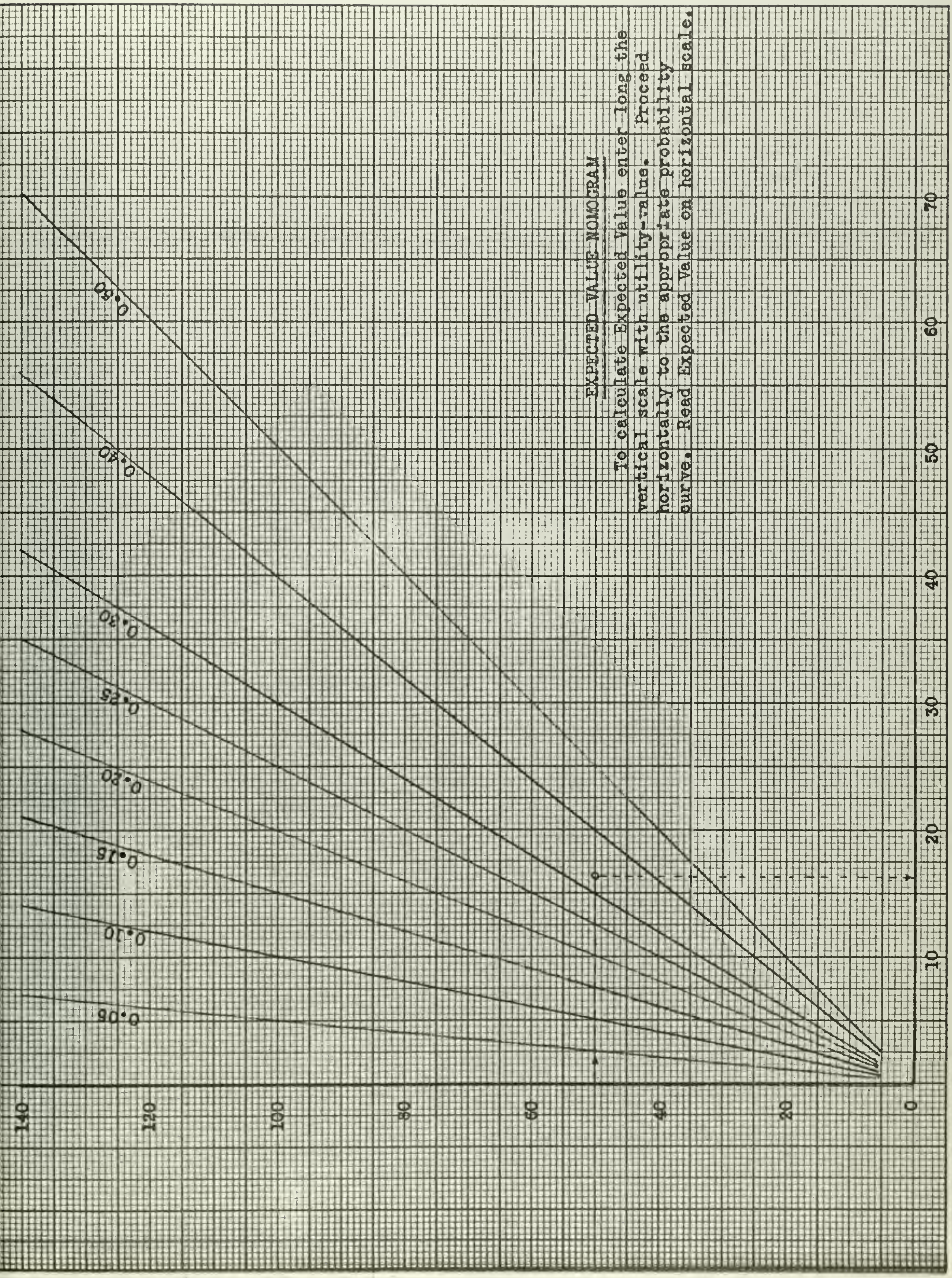

EXPECTED VALUE NOMOGRAM
To calculate Expected Value enter long the vertical scale with utility-value. Proceed horizontally to the appropriate probability curve. Read Expected Value on horizontal scale.
0.50
0.40
0.30
0.25
0.20
0.15
0.10
0.05
140
120
100
80
60
40
20
0
10
20
30
40
50
60
70

FIGURE 9

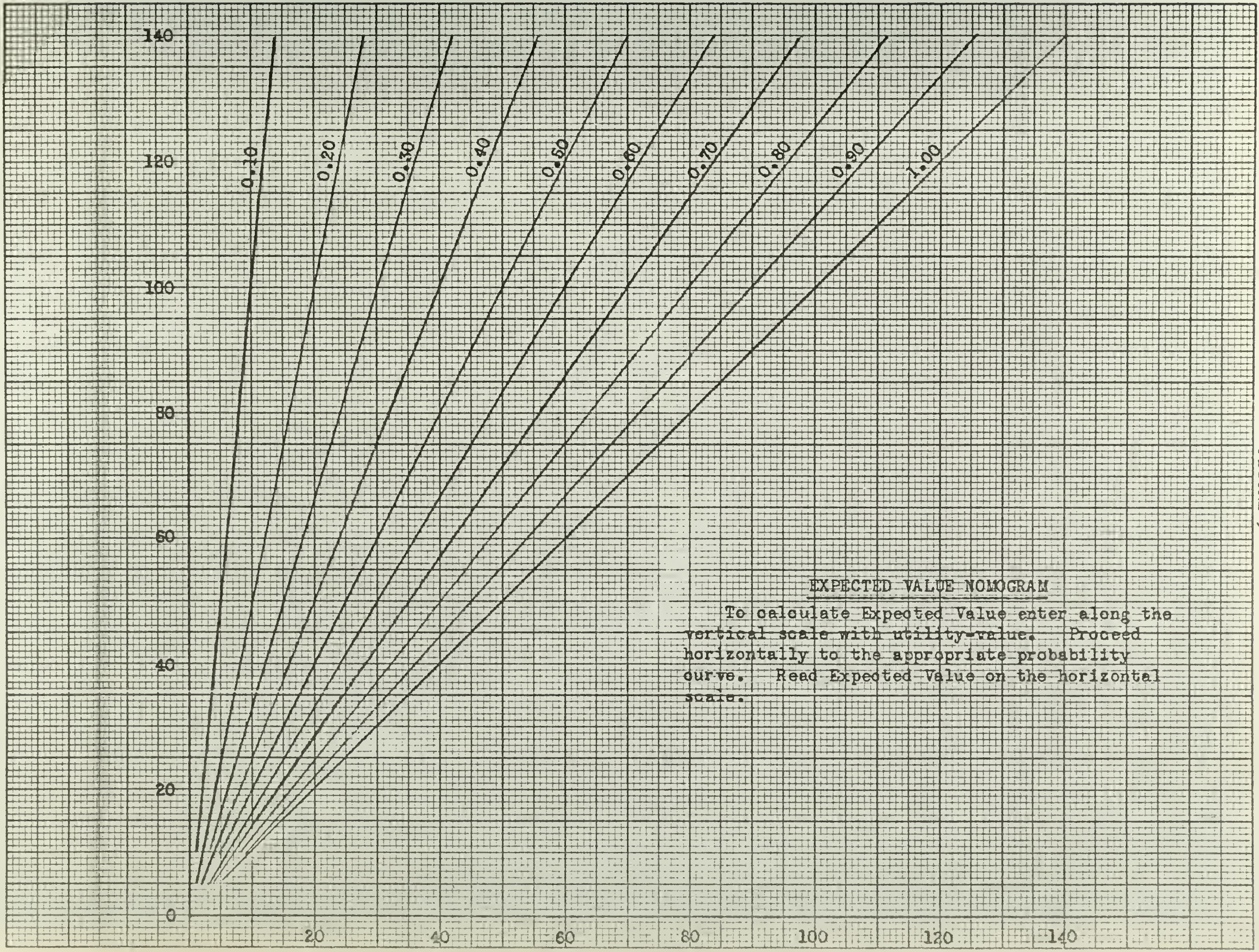
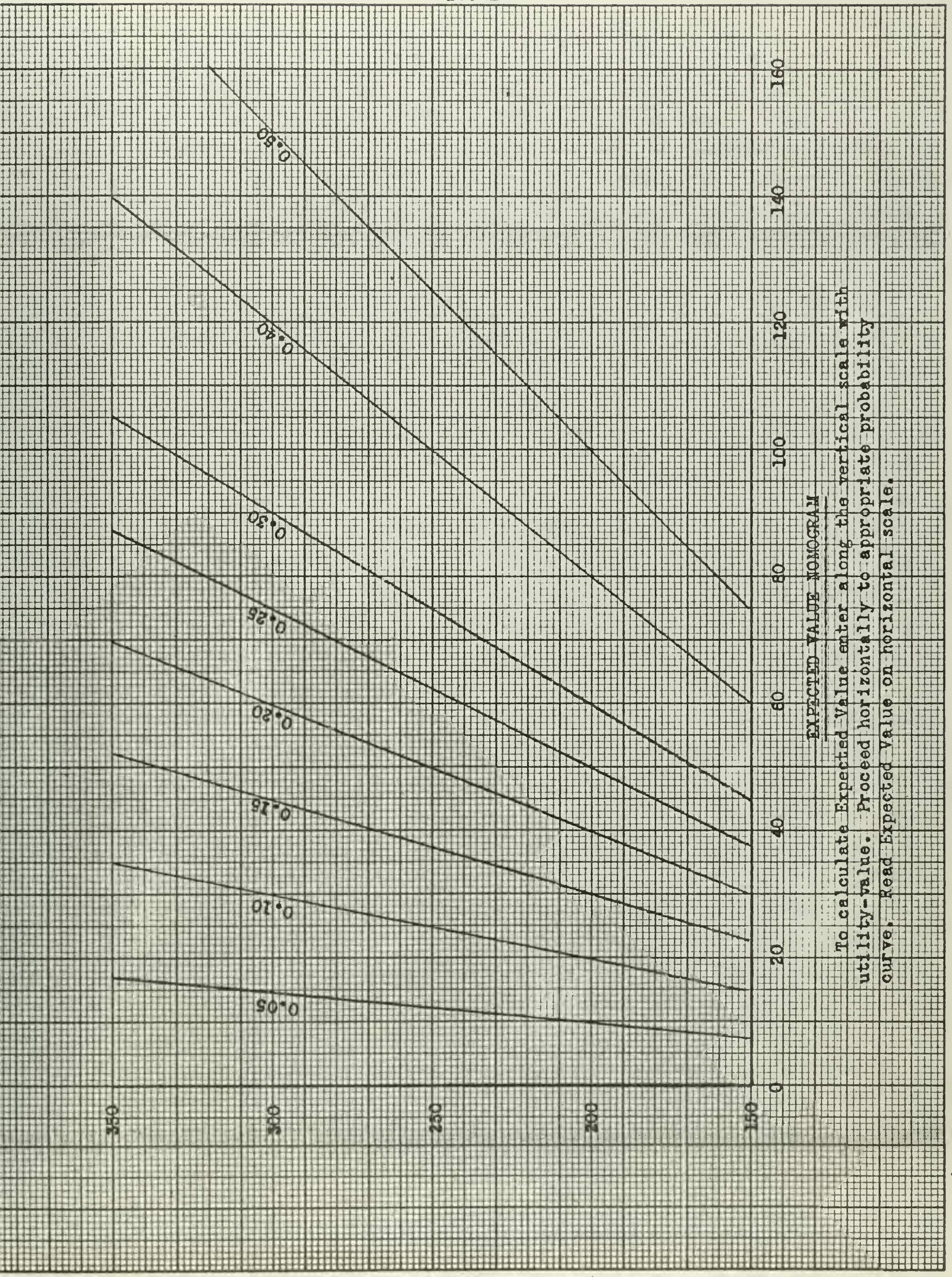

EXPECTED VALUE NOMOGRAM
To calculate Expected Value enter along the vertical scale with utility-value. Proceed horizontally to the appropriate probability curve. Read Expected Value on the horizontal scale.
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
0
20
40
60
80
100
120
140

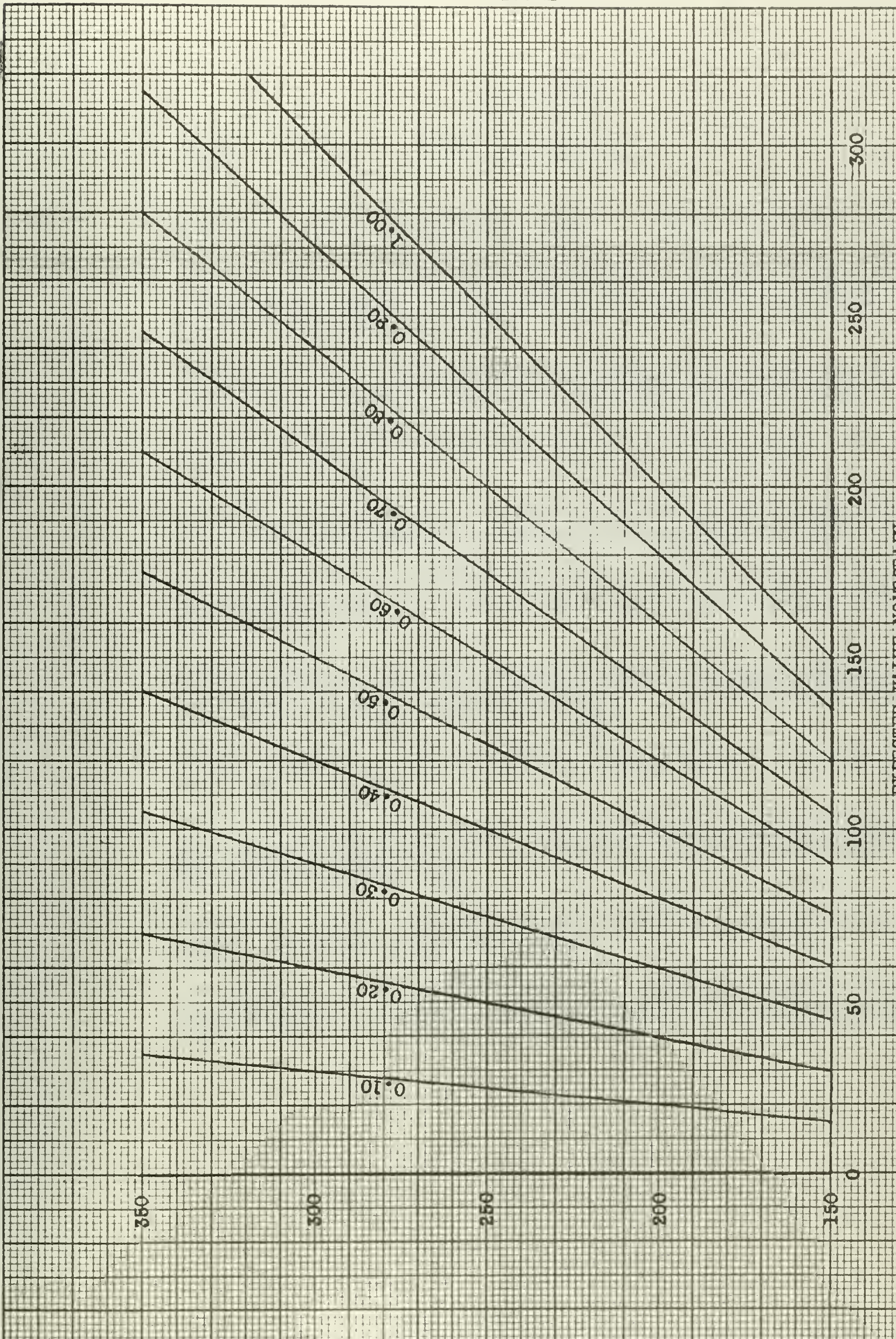
FIGURE 10



EXPECTED VALUE NOMOGRAM

To calculate Expected Value enter along the vertical scale with utility-value. Proceed horizontally to appropriate probability curve. Read Expected Value on horizontal scale.

FIGURE 11



EXPECTED VALUE NOMOGRAPH

To calculate Expected Value enter along the vertical scale with utility-value. Proceed horizontally to the appropriate probability curve. Read Expected Value on the horizontal scale.

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